Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International Advanced Level
In Further Pure Mathematics F1 (WFM01)
Paper 01

WFM02 Report June 2024

Summary

The paper was well received and provided clear opportunities for all grades to access marks, with very many excellent performances seen on the paper. All questions were attempted with no indication of lack of time, and very few incomplete attempts at question 10. There was a significant amount of calculus work on the paper (as it is the large focus of the specification) meaning those with strong skills in this area were able to score highly.

No question caused unexpected trouble and candidates were able to quickly speed through some of the routine topics, such as method of differences or solutions of second order differential equations. A lack of working was seen in some candidates work, with a stress on the importance of showing full workings needing to be reasserted to candidates.

Question 1

This provided most candidates with a good opportunity to score early marks in part (a), though the sketch in part (b) was less secure. Those who were unable to make progress in this question were unlikely to score highly on the paper overall.

Part (a) proved very straightforward to most candidates, though a few did stumble even at this early stage. The two methods shown on the scheme were both seen frequently, though the standard algebraic approach was marginally more popular, and in general successfully completed. Most set up the correct initial equation using the moduli and were able to reach the required equation, though a few slips in simplification did occur in the constant term 23. Some failed to rearrange to the correct form, but this was rare, and only a very few made errors in the complex work by including extra i terms resulting in subtraction of the brackets.

Fewer candidates chose to take the geometric approach, and this approach was the more prone to error. Though this method shows a good understanding of the question, and though most who realised the full method were successful in reaching a correct equation, some made errors in determining the gradient of the line joining $(3,4)$ and $(-1, -1)$ losing accuracy. More serious common errors included forgetting to take the negative reciprocal to find the bisector or omitting find the midpoint at all, losing all three marks in the question.

Part (b) caused far greater trouble, with students unable to see the geometry of the situation. Those who sketched and used the geometric approach to (a) had better success here (even if incorrect in part (a)). Often, however, an incorrect line was drawn, either with positive gradient (the line through the two key points) or crossing the negative imaginary axis. Where a response gave an acceptable line, far too often the wrong side of it was shaded, or only partial shading of a restricted region, or no shading at all. There was perhaps a misunderstanding that the use of "$\leqslant$" implied that they should shade "*below*" the line.

Question 2

Calculus skills were very good throughout the paper and there were very many fully correct responses to this question. Perhaps the given form of the answer helped, as it reminded of the need to use the chain rule when differentiating to generate the $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$. Use of Taylor series was also well understood.

Part (a) was well answered by the majority with most candidates able to use implicit differentiation accurately and having a correct process. The need for the use of both the chain rule and the product rule was well understood – possibly helped by the given form –and candidates were more likely to make errors with rearranging, for instance with sign errors being made, rather than failing to reach the required form. A few did not appreciate the difference between $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$, but most had little trouble in reaching the correct answer. Candidates who opted to rearrange first, using the Alt method, often had less success as the composite of three terms (x, y and $\frac{\mathrm{d}y}{\mathrm{d}x}$) from the first to second derivative was much more involved and candidates had difficulty sorting through it. Again, part (b) was answered well with most getting the correct format though errors from (a) led to some not getting the final accuracy mark. The most common error was to use an incorrect value for y at $x = 2$, despite that this was given in the question, with the initial "1 +…" sometimes missing. All too often, students failed to show working, and simply gave the answer, but the better responses saw candidates calculate values for the differentials clearly first, before later substituting into the general form. The mark scheme was generous in allowing the final mark even if the "y =" was missing, but this should not be relied upon and candidates should be made aware they should give a full answer at the end of their attempts.

Question 3

A very much expected question in which the majority of candidates were able to make quick progress through parts (a) and (b) before having to think a little about how to relate the summation in part (c) to the question. Most were able to fully complete the question and gain full marks.

Part (a) was a very standard partial fractions question and caused no difficulties for the vast majority of candidates. A variety of methods to find the coefficients were seen, but the correct answer was found in almost all cases, and if incorrect answers were found it usually meant little further was even attempted in the question as the telescoping series would fail.

Following completion of the partial fractions question in part (a), candidates easily recognised part (b) as being a standard method of differences problem, which most understood well, showing the first and last few terms to illustrate the process. After setting out the starting and ending terms of the summation, the majority went on to correctly select the non-cancelling ones. Only a small minority neglected to show working in extracting the correct terms, though credit was allowed for the correct terms extracted without further terms seen. The $\frac{1}{2}$ was often included from the outset, but many also realised they could keep it out as a common factor, some ignoring it to start with and only introducing it later in the solution. Combining to a single fraction was generally done well, with most able to combine in one step, while others combined in stages. Whilst the required algebraic expression was often reached, many candidates demonstrated a lack of algebraic fluency by labouring over the solution. However, the question gave the required form of the answer, including the "40" within the denominator which enabled some candidates were able to identify and correct their algebraic errors.

In part (c) most candidates were able to apply their formula from part (b) to achieve the required result by identifying that they needed to use $n = 21$ and $n = 5$, and then subtract the two results. The common error of using $n = 6$ was seen frequently, and $n = 16$ was also seen in a few incorrect responses (i.e. there are 16 terms in the given summation), while limits of $n = 26$ and $n = 8$ were also seen.

It was apparent that a small minority of candidates did not understand how to use their result from part (b). Some students correctly stated the values $n = 21$ and $n = 5$ but without substituting into their formula. Attempts at starting from 0 or going up as far as $n + 1$ also occurred occasionally. A few candidates reiterated the process of differences with the starting and ending terms given, instead of using the answer to part (b). Others simply resorted to using their calculator's summation function to obtain the correct answer. Naturally, this strategy, whilst obtaining the correct answer, is guaranteed to result in no credit being given.

Question 4

The second calculus question of the paper, and the first of two requiring the reduction of a differential equation via a substitution, this was done well by most candidates, though a few did struggle in part (b) to get the final form.

Part (a) was well attempted by most candidates, with the majority able to arrive at a first derivative connecting $\frac{dy}{dx}$ and $\frac{dt}{dx}$. There were a variety of methods seen to achieve this, but the most common way was for candidates to differentiate the given substitution implicitly with respect to x and then rearrange to obtain a correct derivative for $\frac{dy}{dx}$ which could be substituted into equation (I). Rearranging before differentiating was also common, and generally done well. However, some candidates did not find the connection explicitly, but found $\frac{dy}{dt}$ only and implying the correct connection when substituting into the equation.

While most were able to proceed to make the substation correctly, some candidates faced challenges when replacing the y terms with t terms because of unconvincing indices work. Another common approach was to make t the subject and differentiate using the chain rule which yielded many successful attempts, but once again some candidates failed to show their elimination process of the y terms in sufficient detail, losing the final mark. Candidates should be reminded that all steps are required to be shown in a 'show that' question. A few successful attempts were seen in progressing from equation (II) to equation (I), but some candidates' work was hard to follow and could easily have been dismissed as incorrect.

Part (b) was highly successful for well-prepared candidates. Almost all candidates recognised this as an integrating factor type differential equation, and the integrating factor was easily obtained for most. Some carelessness with variables led to some obtaining the integrating factor in terms of t not x which cost them valuable marks in subsequent stages of the question. It was pleasing to see most candidates demonstrate complete competence in integration by parts methods, with sign/coefficient errors being the most common reasons for marks being withheld, sometimes the -2 was forgotten after the integration by parts was separately carried out. A relatively low proportion of candidates failed to multiply the through by the integrating factor correctly, which has often been common amongst the incorrect attempts seen in the past. Treating the constant of integration correctly was usually done well, though there are still many who would neglect to multiply it by the e^{-2x} when rearranging. Reversing the substitution was usually completed correctly, but some poor algebraic skills in reciprocating term by term rather than the whole expression were seen, though such attempts were permitted the method.

It should be noted that only a handful of attempts were seen using the complementary function + particular integral method. It was evident that candidates are well versed in this kind of 'processing' type question.

Question 5

An expected topic, in which most candidates were able to make progress in finding the critical values. A few unprepared candidates struggled to make any progress, but the overall strategy was shown to be known by most.

A variety of approaches was seen, with some realising that finding the critical values by setting the curve equations equal was the necessary first step. This made the algebra required easier and is the most direct approach. Others tried to deal more formally with the inequalities by using the standard approach of multiplying both sides by $(x-3)^2(x+2)^2$, or gathering to one side and combining to a single fraction. Some created extra work by splitting into regions where the linear factors changed signs first, and then repeated the process involved in finding the critical value for each case.

Errors in inequalities did occur when some multiplied through by an incorrect factor such as $(x-3)(x+2)$, but these could still obtain full marks as this would lead to the same critical values.

Most candidates recognised 3 and -2 as critical values, either directly or later from the factorised form, and correctly handled the algebraic requirement to achieve a quadratic expression in x via their chosen method. However, in a few instances where multiplying through and factorising was attempted the $(x+2)$ factor became $(x-2)$ and so the initial B mark was lost.

Once a quadratic was obtained, either correct or incorrect due to incorrect simplification, solving it to find the remaining two critical values was usually done correctly. Being allowed to use a calculator to solve the quadratic clearly helped some candidates. Some students, however, mechanically multiplied everything out to a quartic, therefore requiring them to solve a quartic polynomial. Often this would be done on a calculator and forfeit several marks for failing to use algebra and show all working.

The most challenging part of this question came in the last two marks where, having found four critical values, candidates sometimes showed a lack of understanding as to how to use them to choose the correct regions. Those who drew a sketch, or used a number line, were generally able to pick the correct shape of the regions, but were not always sure when to use the $<$ and $\leqslant$ inequality symbols within their answer, often including the -2 and 3 values as a part of the solution. Some chosen the inner regions instead, while those who omitted the -2 and 3 as critical values (or lost a critical value due to other error) were unable to access the last two marks at all.

Question 6

Again, this was an expected, standard question in which most candidates were very well practised, though less successful than was question 5. Though the method needed for his kind of transformations question was shown by most, there were numerous typical errors made, either through incorrect rearranging, or misinterpretation the final equation.

The main scheme was overwhelmingly the most popular method, with very few attempts at either alternative seen, and Way 2 was usually not successful if attempted. However, some fully correct versions were seen using the perpendicular bisectors method (way 3 on the MS). Another approach observed, with some success was to substitute z into the equation $|z-1|=|z+1|$ for the imaginary axis.

For the main scheme, the majority were able to correctly make z the subject before using complex numbers for w and multiplying by a correct conjugate. However, there was a sizeable proportion who failed in rearranging correctly with $\frac{w\text{i}+\text{i}}{1-w}$ being a surprisingly common incorrect rearrangement. Such candidates could continue with the question, though, but applying the correct process to their fraction. Pleasingly most candidates had a good understanding of what a conjugate was, but some multiplied by expressions that did not generate a real term on the denominator.

Most proceeded to set their real part of the resulting expression equal to 0 to obtain an expression in u and v, though some set the imaginary part to zero instead misunderstanding the geometry of the situation. In some cases candidates arrived at the correct final answer fortuitously through such incorrect work and incorrect multiplication meaning the imaginary and real parts were interchanged, but these could not access the latter marks due to the dependency. Indeed, other fortuitous solutions via an incorrect rearrangement for z and then subsequent incorrect work for their z expression also occurred.

Completing the square was usually well done by those who reached a suitable form, but some candidates did not realise that a circle equation was not obtained if the coefficients of the u^2 and v^2 terms were not equal. Sign errors were highly prevalent amongst the candidates who had a correct method but did not score full marks, usually these arose during the completion of the square.

Even among those who managed to obtain a fully correct equation in u and v and complete the square, not all succeeded in interpreting the centre and radius correctly, with the radius being more troublesome, as errors in the signs of completing the square, or neglecting to square root led to incorrect answers. Notation for the centre was sometimes incorrect, though tolerance was given on answers such as $\left(\frac{1}{2},-\frac{1}{2}\text{i}\right)$, but candidates should ensure they are clear as to the difference between Cartesian coordinate and complex numbers.

Question 7

Third of the five calculus related questions, again this was very well answered with candidates showing strong differentiation skills throughout, and the Maclaurin expansion being done well.

Part (a) saw most pick up the first three marks with relative ease, applying the product rule successively to reach the fourth derivative. Most went on to find the correct value for *k* by continuing to the sixth derivative, though a few went awry in simplification following a correct fourth derivative (such as combining $-2\sin x - 2\sin x$ to $-2\sin x$), which ultimately resulted in incorrect 5th and 6th derivatives. A few did not simplify their work as they went along, making later differentials unnecessarily complicated. There were also numerous cases of fully correct differentiation up to the sixth derivative followed by an incorrect final answer given, usually either losing the negative sign, or mistakenly giving -8 as the value for *k*.

The vast majority worked exclusively in terms of *x*, not spotting that substitution for *y* at the fourth derivative made the latter derivatives easier. In contrast, some attempted to substitute for *y* at early stages leading to more complicated expressions in *x* and *y*. However, these were still able to proceed to the correct answer.

Part (b) was a fairly routine part but did lead to some surprising errors due to the number of terms that needed to be evaluated. Not all produced went to the 6th power term, giving only four non-zero terms, though some had a non-zero *y* or $\frac{d^4y}{dx^4}$ meaning they obtained five non-zero terms before needing the sixth derivative. Such candidates would have done well to question why the sixth derivative was asked for in part (a).

Once values were found for the derivatives, the Maclaurin series was usually generated correctly, but many students did not explicitly show the values of derivatives first, instead producing the series in situ. Usually the factorials were shown to demonstrate the correct formula, but in some cases correct answers implied the marks. Warning should be reiterated that incorrect answers without the formula clear would result in the forfeiture of marks. Candidates are advised to first find the values of the derivatives at $x = 0$, and only then go on to substituting them to Maclaurin's formula, rather than trying to do everything in one go.

A surprisingly common slip was for a student to use $y_0 = 1$, presumably think that $e^0 = 1$ and forgetting the sine term. This is one way in which they ended series at the x^5 term, as noted above.

A few candidates did not follow the "Hence" instruction of the question and attempted to multiply the exponential and trigonometric Maclaurin series together. This was rarely successful in practice, and never successful in gaining any credit.

Question 8

Following on from question 7, a return to differential equations in the fourth calculus question again saw a good demonstration of skills in calculus but the majority of candidates. Though a little less successful with obtaining the second derivative correctly, the given result and given equation (II) meant that progress through the rest of the question was not hampered.

A varied mix of responses was seen part (a), to least well answered part of the question overall. Candidates nevertheless were generally very successful in this part and were able to connect $\frac{dy}{dt}$ and $\frac{dy}{dx}$ before proceeding to connect second derivatives and show good understanding of the principles required for transforming equations. The chain rule was generally well used when it was employed though the most common error in this part was to omit its use and introduce the extra e^{-t} term just to make the question work. A few errors in reciprocals of the derivatives also occurred in some cases where the chain rule was attempted, but with the answer given, candidates somehow found a way to reach it, rather than checking back to find errors when it clearly did not follow their previous work.

Part (b) was very routine, and the mark scored by the vast majority for the rather simple substitution of the give result into the equation (I). A few who had gone wrong in part (a) did not attempt this part at all, the most did realise they could use the given result. Having an overall awareness of the question structure would be advisable. Where attempts were made that did not score the marks, errors with using x instead of t for the powers was the most usual reason.

Once into part (c) candidates showed much familiarity with the process involved, with the correct quadratic being solved successfully almost exclusively. The complementary function was sometimes given as a function of x rather than as a function of t, but the form was shown usually correct and, in many cases, the correct variable was recovered later (though if not the accuracy mark was lost). The required form for the particular integral was known by nearly all candidates, and the process required to find the constants was well versed and there were many completely correct solutions. Occasionally slips in solving the equations did occur result in the loss of the final accuracy mark but the formation of the final solution by adding the complementary function and particular integral was again well understood, and the correct "$y =$" at the start usually given.

Part (d) was again routine for most, though some candidates did not respond to this part, perhaps thinking they already had the solution. Those that did answer could score the follow through mark even if they had been incorrect in part (c). However, many did not simplify the exponentials fully, and so did not score the mark. Labelling of the function was usually correct in part (c) and (d), so few lost a mark for not naming the function correctly. Another error in this part seen numerous times was incorrect simplification of the log term $(\ln x)^2$ to $2\ln x$ without showing an intermediate step to award the mark for.

Question 9

This question tested the candidate’s ability to accurately complete the steps of a De Moivre’s theorem proof and then apply the result to solve and equation. The processes for both parts were demonstrated well, but fully details were not always given in part (a), while many did not heed that the smallest positive root was asked for in part (b) and gave the first root found instead.

For part (a) the vast majority used the intended main scheme and applied De Moivre’s theorem successfully to find an expression for cos 6θ by extracting the real terms. Not all showed the cos 6θ but the expansion of the brackets via the binomial theorem was generally correct (only a few expanded in stages). Having extracted the real terms, use of the Pythagorean identity to get to an expression in cos θ only was carried out almost always, with only a few making slips at this stage, and the first three marks were gained in most cases. The final mark proved more challenging as all the steps needed to be correct, and many did not show the full expansions or made errors with the expansion or neglected the cos 6θ throughout the whole of their proof.

The alternative method for part (a), via $\left(z+\frac{1}{z}\right)^6$, had similar performance as the main scheme when attempted.

Part (b) similarly saw many score the first three marks with relative ease but only a minority scored full marks as many candidates assumed the smallest solution was the one that came from the principal value for 6θ. Others had an incorrect value for $\cos 6\theta$ from the outset, with those assuming $\cos 6\theta = 0$ unable to access any marks. Those that did have a non-zero value for $\cos 6\theta$ sometimes did not proceed correctly, or at all, to a value for θ (some only proceeded as far as 6θ). Many opted to take the positive of their negative value and find the angle in the first quadrant but did not remember to convert to the correct quadrant afterwards. There was a sizeable proportion, as well, to reach a value for θ correctly but failed to undo the substitution to reach a value for x, as well as the many who achieved the 0.950 answer for x but did not consider if there were smaller positive values.

For those who achieved the correct answer, many found all 6 values for x first, before selecting the smallest positive one, producing quite a lot of effort for one mark. Those who focussed directly on the smallest value most likely were using their calculators to work it out, in order to give only the necessary solution, and in some cases the working didn’t always fit the final answer as the correct intermediate value for θ was not given. Some resorted to calculator use entirely, giving only the answer from a graphical means, while others solved the cubic in x^2 directly on the calculator to find the roots, not using part (a) at all, contravening the “hence” of the question and scoring no marks. Overall, this part of the question was therefore a little unsatisfactory in terms of performing as intended as the most diligent answers took more effort for no greater reward than those who used calculators to identify the correct solution and showed minimal working to justify it.

Question 10

The final question on the paper was well placed, but still provided good access to all candidates. More calculus work was required for part (b), but the process of integrating $\cos^2\theta$ in a polar integral is a well-versed topic that provided little challenge. The greater difficulty came in seeing how to build up the required area from the constituent parts, with incorrect limits being used, or incorrect attempts at the triangle being the main sticking points.

Part (a) was a little different to what candidates have seen in the past, but most dealt with this task relatively easily. Setting the $1+\cos\theta = OP$ to first find θ, then using this in $r = k\sec\theta$ was the most common approach here. Consequently, it was common to see the initially B mark for part (b) scored at this point, as this was obtained in a natural attempt to find the value of *k* in part (a). Very few indeed spotted that $\cos\theta$ could be eliminated from the equations without the need to find θ first, but it amounted to the same thing. A few were not able to make progress in this part as they tried to set the two equations equal, $1+\cos\theta = k\sec\theta,$ and tried to solve a quadratic in $\cos^2\theta$ and *k* but could not do so. Some did realise the need to use *OP* afterwards, but many who did this simply stopped. Since *k* was not necessary to proceed in part (b) this did not stop further progress in the question.

Part (b) was standard Further Mathematics territory, and many candidates scored at least the two marks in (b) for a correctly integrated expression for the polar curve, as well as the B mark that was commonly already scored for work in part (a). Only a very small proportion of candidates did not square the equation for *r* correctly or obtained only two terms. Attempts using incorrect double angle identities were also relatively rare, further demonstrating that candidates are well practised in this kind of question.

Errors with limits did occur frequently, as did errors in evaluating the area of the triangle, but candidates generally showed they know the holistic process needed to find the area inside a polar curve and bounding lines. The most common error in limits was to integrate between 0 and π or $\frac{\pi}{2}$, though other combinations were seen (such as $\frac{\pi}{2}$ to π). Such attempts often still attempted the triangle but were unable to score the mark for the full process. For those using correct limits, errors in the substitution or slips in accuracy evaluating were rare.

Using $\frac{1}{2}\times r\sin\theta \times r\cos\theta$ was the most common way seen for the area of the triangle, while using integration to attempt it was also relatively common but proved to be more challenging and so more prone to accuracy error. However, most were able to identify a correct method to obtain the area. Errors in combining the areas incorrectly led to some candidates forfeiting the final marks, but generally those who had found the constituent pieces would add the two areas, so success depended on the methods for the two areas.

Overall, the question was very well answered with many fully correct responses, indicating there were no timing issues hampering the prepared candidates.